Bolt: Anonymous Payment Channels for Decentralized Currencies

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ABSTRACT

Bitcoin owes its success to the fact that transactions are transparently recorded in the blockchain, a global public ledger that removes the need for trusted parties. Unfortunately, recording every transaction in the blockchain causes privacy, latency, and scalability issues. Building on recent proposals for "micropayment channels" — two party associations that use the ledger only for dispute resolution — we introduce techniques for constructing anonymous payment channels. Our proposals allow for secure, instantaneous and private payments that substantially reduce the storage burden on the payment network. Specifically, we introduce three channel proposals, including a technique that allows payments via untrusted intermediaries. We build a concrete implementation of our scheme and show that it can be deployed via a soft fork to existing anonymous currencies such as ZCash.

1 INTRODUCTION

Bitcoin has become increasingly popular as a decentralized electronic currency. In Bitcoin, each transaction is recorded in the blockchain, a public transaction ledger maintained by a set of decentralized peers. While this design has proven successful at low transaction volumes, the reliance on a globally-shared ledger has caused serious scaling issues. Since in Bitcoin 1MB blocks are added to the blockchain every ten minutes on average, the Bitcoin transaction rate is limited to fewer than ten new transactions per second across the entire Bitcoin user base [1].\(^1\) Several proposals to increase blockchain bandwidth are being debated in the Bitcoin community today, but none are likely to produce a transaction rate that competes with centralized services such as payment card networks.

A promising approach to addressing the scaling problem is to move the bulk of Bitcoin transactions off chain, while preserving the system’s decentralized structure and strong integrity guarantees. The leading proposal for off-chain payments is to use payment channels, exemplified by the Lightning Network [45] and Duplex Micropayment Channels [30]. Rather than posting individual payment transactions to the blockchain, channels employ the blockchain to first establish a shared deposit between two parties. The parties interact directly to make payments — adjusting the respective ownership shares of the deposit — and communicate with the blockchain only to agree on the final split of escrowed funds. In cases where no direct payment channel exists between two parties, these proposals also allow participants to route transactions via intermediate peers [45]. The main benefit of the payment channel paradigm is that it dramatically reduces the transaction volume arriving at the blockchain, without adding new trusted and centralized parties.

While payment channels offer a solution to the scaling problem, they inherit many of the well-known privacy weaknesses of Bitcoin [40, 46]. Although payments are conducted off chain, any party may learn the pseudonymous identities and initial (resp. final) channel balances of the participants. More critically, payment channels provide very limited privacy against transaction counterparties. By establishing a channel to pay for e.g., Tor bandwidth or web content, a user implicitly links each payment on a given channel to all of her other payments on this channel. This is particularly problematic in the likely event that payments are routed via a common intermediate peer — such as a currency exchange — since the intermediary must now be trusted to keep private your full payment history. Some proposals, such as the Lightning Network, have proposed to work around this problem by routing the payment via multiple intermediary nodes; however (as we discuss in §6) this approach substantially increases the complexity of establishing payment channels, and reveals payment information in the event that even a subset of the intermediaries collude.

Several academic works have recently proposed solutions that address the privacy problems of Bitcoin-type currencies [29, 41, 42, 47]. Some of the resulting systems been publicly deployed, notably ZCash [3] (an implementation of the Zerocash protocol [47]) and Monero [2]. Unfortunately, the privacy mechanisms contained in these systems apply to the privacy of transactions on the blockchain, and do not address the setting of payment channels. Indeed privacy for payment channels seems fundamentally challenging due to channels’ pairwise structure. Even when a channel is funded with anonymous currency, repeated payments within the same channel are inherently linkable. This is concerning, given that one of the main proposed applications of channels is for web micropayments — which are often described as a more private alternative to tracking and online behavioral advertising.

We stress that concerns about privacy are not theoretical. Several commercial ventures [11, 23, 32] have been founded around the task of analyzing and tracing blockchain transactions. It is reasonable to expect that surveillance will be applied to payment channel systems if they become widely deployed.

Our Contribution. In this paper we propose Blind Off-chain Light-weight Transactions, or Bolt. Bolt consists of a set of techniques for

\(^1\)As of early May 2017, this has resulted in a backlog of nearly 165,000 transactions [15].

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constructing privacy-preserving payment channels for a decentralized currency. These techniques ensure that multiple payments on a single channel are unlinkable to each other and — if channels are funded with anonymized capital \(^2\) — anonymous.

Our constructions enhance earlier work in privacy-preserving decentralized payments \([29, 41, 47]\) while addressing the problem of providing fast and private off-chain transactions. Unlike earlier proposals \([37]\), which simply obfuscate participant identities from intermediaries, our proposals create anonymous direct channels even when a merchant does not know the identity of the paying party. Of more practical interest, we prototype one of our constructions and show that we can deploy Bolt as a soft fork of a Bitcoin like cryptocurrency. We provide constructions for three types of channels:

- **Unidirectional payment channels** allow a customer to pay a merchant fixed value coins repeatedly without linking the payments together. While our approach builds on the compact e-cash paradigm introduced by Camenisch et al. \([17]\), it requires a novel mechanism to achieve succinct channel closure, ensuring blockchain space usage is constant, regardless of the transaction volume.

- **Bidirectional Channels** allow two parties to exchange arbitrary valued payments in either direction, without linking the payments. The challenge here is preventing a malicious counterparty from using obsolete information to claim an earlier balance, while maintaining the scheme’s unlinkability. While multiple payments on the same channel are unlinkable, to avoid linking an aborted payment to the payer’s identity, our construction requires that the underlying payment channel be funded anonymously.

- **Indirect channels** Finally, we extend our bidirectional payment channel construction to enable third party payments, where an untrusted intermediary acts as a “bridge” allowing two otherwise unconnected parties to exchange value. Critically, the intermediary learns neither the identity of the parties nor the amount transacted.

### 1.1 Background on Payment Channels

A payment channel is a relationship established between two participants in a privacy-preserving decentralized ledger-based currency network. While payments may flow in either direction on an established channel, the parties themselves are not symmetric: for a payment channel to work, at least one party must initiate the connection. For simplicity of exposition, we will refer to the initiating party as a customer, and the responding party as a merchant. We assume that the payment network includes a means to validate published transactions and to resolve disputes according to public rules. In principle these requirements can be satisfied by the scripting systems of consensus networks such as Monero or ZCash, using only minimal script extensions (which we discuss in §5.) We stress that our proposals in this work focus on the privacy of payment channels, and thus we assume the privacy of the underlying funding network.

When two parties wish to open a channel, the parties first agree on the respective balance shares of the channel, which we represent by non-negative integers \(\beta_{\text{merch}}^i\) and \(\beta_{\text{cust}}^i\). The parties establish the channel by posting a payment to the network. Provided that these transactions are correctly structured, the network places the submitted funds in “escrow” until a subsequent closure transaction is received. The customer now conducts payments by interacting off-chain with the merchant. For some positive or negative integer payment amount \(\epsilon_i\), the \(i\text{th}\) payment can be viewed as a request to update \(\beta_{\text{merch}}^i := \beta_{\text{merch}}^{i-1} + \epsilon_i\) and \(\beta_{\text{cust}}^i := \beta_{\text{cust}}^{i-1} - \epsilon_i\), with the sole restriction that \(\beta_{\text{merch}}^i \geq 0\) and \(\beta_{\text{cust}}^i \geq 0\). At any point, one or both parties may request to close the channel by posting a channel closure message to the ledger. If the closure messages indicate that the parties disagree about the current state of the channel, the ledger executes a dispute resolution algorithm to determine the final channel balances. After a delay sufficient to ensure each party has had an opportunity to contribute its closure message, the parties may recover their final shares of the channel balance using an on-chain payment transaction.

Any payment channel must meet two specific requirements, which we refer to as universal arbitration and succinctness:

1. **Universal arbitration.** In the event that two parties disagree about the state of a shared channel, the payment network can reliably arbitrate the dispute without requiring any private information.

2. ** Succinctness.** To make payments scalable, all information posted to the ledger must be compact — i.e., the size of this data should not grow linearly with the balance of the channel, the number of transactions or the amounts exchanged.

The latter property is an essential requirement for the setting of payment channels, since it rules out degenerate solutions that result in a posted transaction for every offline payment, or that post the full off-chain payment interaction to the ledger.

### 1.2 Customers, Merchants, and the Limits of Anonymity for Payment Channels

Informally our constructions for payment channels provide the following privacy guarantee:

> Upon receiving a payment from some customer, the merchant learns no information beyond the fact that a valid payment (of some known positive or negative value) has occurred on a channel that is open with them. The network learns only that a channel of some balance has been opened or closed.

Note, however, that the privacy protections against a channel participant are slightly weaker than those against third parties. This is an inherent limitation of the payment channel setting. Moreover, these limitations change depending on if a payment is made over a single direct channel or an indirect channel consisting of a series of channels between the customer, one or more intermediaries, and a merchant. We explain these limitations further here.

**Direct channels.** The direct channel setting has three limitations. First, the privacy provided for direct channels is asymmetric: only

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\(^2\) This requires either that the underlying cryptocurrency is itself anonymous, e.g. as in ZCash \([3]\), or that there exists some way of anonymizing or mixing that adds sufficient anonymity to non-anonymous cryptocurrency.
the party initiating the payment is anonymous and unlinkable between payments, while the target of the payment is pseudonymous. This holds because at least one party must know which payment channel is being used.

Second, when receiving a payment on a channel, the recipient knows the payment came from someone with whom they have an open channel. This is also fundamental to the nature of channels, since they must be established with a counter-party before being used.

This final requirement suggests that recipients should use well-known channel parameters to group all channels, and thus maximize the anonymity set of its customers. If a recipient provides unique channel parameters to each potential payer (i.e., behaves as though it was a different party to each payer), then the payer receives no privacy — as the set of channels open under that set of parameters has an anonymity set of a single person.

This setting maps well to a situations where the payment target is known, e.g., where a single merchant or website accepts payments from many anonymous customers. Thus for the remainder of the paper we term this well-known target party the merchant, and refer to the paying party as the customer. We keep this terminology even when in settings where payment amounts are negative (resulting in a reverse payment), since one party must still be well known and this terminology maps to the case where, e.g., a merchant is refunding a customer for a previous purchase. We stress that to anyone not a party to the payment channel, privacy is absolute.

**Indirect channels** For indirect channels which involve one or more intermediate channels, the privacy guarantees may, surprisingly, be stronger than the direct case. First, this configuration facilitates a larger anonymity set, since it encompasses any party who has a channel open with the entry intermediary (for the initiator) as well as anyone who has a channel open with the exit intermediary (for the target). Additionally, when channels contain a single intermediary, they can be configured such that the merchant remains anonymous to the customer. Specifically, although the underlying pair-wise channels still offer asymmetric privacy (i.e., one party is well-known), we can arrange the indirect channels so that the customer and merchant are both holding the private end of their channel and instead the intermediary is the only well-known party. We discuss this arrangement in §4.3.

The advantages of intermediaries do not fully generalize to chains containing more than a single intermediary. Specifically, we show that channels with a single intermediary can be configured to hide the payment amount from the intermediary. However, channels which involve more than one intermediary cannot hide the value of a payment from all intermediaries. Regardless of cryptographic underpinnings, at least one endpoint of each channel must know the channel balance or else the channel cannot be closed. As a result, in any chain of channels with multiple intermediaries, at least one channel will have an intermediary party on both endpoints, and one of these parties will inevitably learn the value of the payments. This is not a limitation of our techniques but simply a consequence of the nature of payment channels.

1.3 Overview of our constructions

In this work we investigate two separate paradigms for constructing anonymous payment channels. Our first construction builds on the electronic cash, or e-cash, paradigm first introduced by Chaum [25] and extended in many subsequent works, e.g., [13, 17, 26]. This unidirectional construction allows for succinct payments of fixed-value tokens from a customer to a merchant, while preserving the anonymity and functionality of a traditional payment channel. Our second construction extends these ideas to allow for variable-valued payments that traverse the channel in either direction (i.e., each payment may have positive or negative value), at the cost of a more complex abort condition. Finally, we show how to extend our second construction to support path payments where users pay anonymously via a single untrusted intermediate party or a chain of intermediaries.

We now present the intuition behind our constructions.

**Unidirectional payment channels from e-cash.** An e-cash scheme is a specialized protocol in which a trusted party known as a bank issues one-time tokens (called coins) that customers can redeem exactly one time. “Offline” e-cash protocols seem like a natural candidate for implementing a one-way payment channel. For purposes of exposition, let us first consider a “strawman” proposal based on some ideal offline e-cash scheme that allows for the detection of doubly-spent coins. In this proposal, the merchant plays the role of the bank. After confirming that the customer has funded a channel, it issues a “wallet” of anonymous coins to the customer, who then spends them back to the merchant. To close the channel, the customer spends the remaining coins to herself and posts the evidence to the payment network. The merchant can dispute the customer’s statement by providing evidence of a doubly-spent coin.

This strawman protocol suffers from several weaknesses. Most obviously, it is not succinct, since closure requires the customer to post all of her unspent coins. Secondly, there is an issue of timing: the merchant cannot issue a wallet to the customer until the customer’s funds have been escrowed by the network, a process that can take from minutes to hours. At the same time, the customer must be assured that she can recover her funds in the event that the merchant fails to issue her a wallet, or aborts during wallet activation. Finally, to avoid customer “framing” attacks (in which a merchant issues coins to itself and then accuses the customer of double-spending) we require an e-cash scheme with a specific property called **exculpability**: namely, it is possible for any third party (in our case the payment network) to distinguish "true" double spends — made by a cheating customer — from false double-spends created by the merchant.

**Intuition behind our unidirectional construction.** To address the first concern, we begin with a compact e-cash scheme [17]. Introduced by Camenisch et al, this is a form of e-cash in which B separate coins can be generated from a constant-sized wallet stored at the customer (here B is polynomial in the wallet size). While compact e-cash reduces the wallet storage cost, it does not immediately give rise to a succinct closure mechanism for our channels. The key innovation in our construction is a new mechanism that reduces channel closure to a single fixed-size message — at the cost of some increased (off-chain) interaction between the merchant and customer.
To create a payment channel in our construction, the customer first commits to a set of secrets used to formulate the wallet. These are embedded within a succinct wallet commitment that the customer transmits to the payment network along with the customer’s escrowed funds (and an ephemeral public signature verification key \( pk_c \)). The customer and merchant now engage in an interactive channel establishment protocol that operates as follows. The customer first generates \( B \) coin spend transactions, and attaches to each a non-interactive zero knowledge proof that each coin is tied to the wallet commitment. She then individually encrypts each of the resulting transactions using a symmetric encryption scheme such that each ciphertext \( C_j \) embeds a single spend transaction, along with the decryption key for ciphertext \( C_{j+1} \). After individually signing each of the resulting ciphertexts using her secret key, the customer transmits the signed results to the merchant for safekeeping. A critical aspect of this scheme is that from the merchant’s perspective these ciphertexts are opaque: the customer does not need to prove to the merchant that any ciphertext is well-formed.

When the customer wishes to close an active channel with remaining balance \( N \) (for \( 0 < N \leq B \)), she computes \( j = (B - N) + 1 \) and posts a signed message (channel ID, \( j \), \( k_j \)) to the network, with \( k_j \) being the decryption key for the \( j^{th} \) ciphertext. The merchant can use this tuple to decrypt each of the ciphertexts \( C_j, \ldots, C_N \) and thus detect further spending on the channel. If the customer cheats by revealing an invalid decryption key, or if any ciphertext decrypts to an invalid coin, or if the resulting transactions indicate that she has double-spent any coin, the merchant can post indisputable evidence of this cheating to the network — which, to punish the customer, will grant the full channel balance to the merchant.

**Bidirectional payment channels.** A restriction on the previous construction is that it is unidirectional: all payments must flow from the customer to the merchant. While this is sufficient for many useful applications — such as micropayments for web browsing — some applications of payment channels require payments to flow from the merchant to the customer. As we further discuss below, a notable example of such an application is third party payments, where two parties send funds via an intermediary, who must increase the value of one channel while decreasing the other.

For these applications, we propose a second construction that combines techniques from existing (non-anonymous) payment channels with blind signatures and efficient zero-knowledge proofs. As in the existing payment channel systems \([30, 45]\), the customer and merchant first on agree on an initial channel state, with the customer holding \( B_{0}^{\text{cust}} \) escrowed funds, and the merchant provides a signature on this balance. When the customer wishes to pay the merchant an arbitrary positive or negative amount \( v \), she conducts an interactive protocol to (1) prove knowledge of the previous signature on the current balance \( B_{i-1}^{\text{cust}} \), and (2) demonstrate that she possesses sufficient balance to complete the payment. She then (3) blindly extracts a new signed refund token from the merchant containing the updated balance \( B_{i}^{\text{cust}} = B_{i-1}^{\text{cust}} - v \). At any point, the customer may post her most recent refund token to the blockchain to redeem her available funds. See figure 1.

The main challenge in this approach is to prevent a dishonest customer from retaining and using earlier versions of her refund token on channel closure. To prevent this, during each payment, the customer interacts with the merchant to present a revocation token for the previous state. As long as the customer behaves honestly, this revocation token can never be linked to the channel or to any previous transactions. However, if the customer misbehaves by posting an obsolete refund token, the merchant can instantly detect this condition and present the revocation token to the network as proof of the customer’s malfeasance — in which case, the network awards the balance of the channel to the merchant. Unlike the e-cash approach, this proposal suffers from the possibility that one of the parties will abort the protocol early; we address this by using the network to enforce fairness.
From direct to third-party payments. As the concluding element of our work, we show how a bidirectional payment channel can be used to construct third-party payments, in which a first party $A$ pays a second party $B$ via a common, untrusted intermediary $I$ to which both parties have previously established a channel. In practice, this capability eliminates the need for parties to maintain channels with all of their peers. The key advantage of our proposal is that the intermediary $I$ can link transactions to individual users, nor — surprisingly — can they learn the amount being paid in a given transaction. Similarly, even if $I$ is compromised, it cannot claim any transactions passing through it. This technique makes anonymous payment channels usable in practice, provided there exists a highly-available (untrusted) intermediary to route the connections. We provide the full details of our construction and how to extend it to support multiple intermediaries in §4.3.

Aborts. Our unidirectional protocol provides privacy guarantees that are similar to the underlying e-cash protocol, with the obvious (and necessary) limitation that final channel balances are revealed on closure. Payments between a customer and merchant are non-interactive and completely anonymous. The bidirectional payment construction, on the other hand, provides a slightly weaker guarantee: by aborting during protocol execution, the merchant can place the customer in a state where she is unable to conduct future transactions. This does not prevent the customer from resorting to the network to close the channel, but it does raise concerns for anonymity in two ways:

1) The merchant can arbitrarily reduce the anonymity set by (even temporarily) evicting other users through induced aborts.

2) The merchant may link a user to a repeating sequence of transactions by aborting the user in the middle of the sequence.

For many traditional commerce settings, the consequences of such aborts may be minimal: no matter the payment mechanism, the merchant can fail to deliver the promised goods and the customer will almost certainly abort. For other settings, such as micropayments, these possibilities should be considered. In such settings customers should scan the network for premature closures and abort the channel if the number of open channels with a merchant falls below their minimal anonymity set.

1.4 Comparison to related work
In concurrent work, Heilman et al. proposed an elegant mixing system called Tumblebit [36]. Tumblebit is compatible with classical Bitcoin and operates in two modes. The first allows users to anonymize (aka mix or “launder”) their own coins. The second mode allows for payment channels between distinct users. Overcoming the limited choice of cryptographic primitives to get Bitcoin compatibility is a serious achievement, but for Tumblebit it comes at the cost of far more limited features, performance, and privacy in comparison to Bolt’s payment channels.

Most significantly, in a payment from Alice to Bob, “Bob and the Tumbler can collude to learn the true identity of Alice” [36] since Alice identifies herself to the Tumbler when making a payment because she must pay the Tumbler with traceable Bitcoins.\footnote{This is likely fundamental. See Section 7.c of [36] for further discussion. Although Heilman et al. provide some mitigations for these attacks, as they acknowledge the Tumbler and Bob can still correlate Alice’s interactions. Thus they cannot offer Alice provable privacy from Bob.} In contrast both our schemes provide provable privacy for Alice even in the face of corrupted and colluding parties. Second, Tumblebit does not hide payment values.

On the functionality side: Tumblebit payments are of a single fixed value and payment channels are unidirectional. In contrast we provide for bidirectional payment channels with variable valued payments. Tumblebit payments are also not succinct: a channel allowing $n$ payment needs either $O(n)$ state on the blockchain or $O(n^2)$ invocations of their protocol.

On the performance side: at 387ms per channel payment, Tumblebit is 5 times slower than our prototype implementation of Bolt’s bidirectional channels. We stress that this is not due to a design flaw in Tumblebit: working within the confines of Bitcoin compatibility is extremely challenging and comes at a high cost.

Finally, like Tumblebit, our unidirectional protocol provides full protections from aborts. Our bidirectional protocol does not and requires an underlying anonymous currency for safety (see §1.3). Variable payments seem to require multiple rounds of interaction, thus risking aborts terminating in invalid intermediate states.

1.5 Outline of this paper
The remainder of this paper proceeds as follows. In §2 we present definitions for anonymous payment channels. In §3 we present the building blocks of our scheme. In §4 we describe the protocols for our payment channel constructions, and in §5 we present concrete instantiations of these protocols. Finally, in §6 we discuss the related work.

2 DEFINITIONS

Notation: Let $\lambda$ be a security parameter. We write $P(\mathcal{A}(a), \mathcal{B}(b)) \rightarrow (c, d)$ to indicate a protocol $P$ run between parties $\mathcal{A}$ and $\mathcal{B}$, where $a$ is $\mathcal{A}$’s input, $c$ is $\mathcal{A}$’s output, $b$ is $\mathcal{B}$’s input and $d$ is $\mathcal{B}$’s output. We will define $\nu(\cdot)$ as a negligible function. We will use $\nu_{\text{max}}$ to denote the maximum balance of a payment channel, and denote by the set of integers $\{\epsilon_{\text{min}}, \ldots, \epsilon_{\text{max}}\}$ the range of valid payment amounts.

2.1 Anonymous Payment Channels
An Anonymous Payment Channel (APC) is a construct established between two parties that interact via a payment network. In this section we first describe the properties of an anonymous payment channel scheme, which is a collection of algorithms and protocols used to establish these channels. We then explain how these schemes can be used to construct channels in a payment network. We now provide a formal definition of an APC scheme.

Definition 2.1 (APC scheme). An anonymous payment channel scheme consists of a tuple of possibly probabilistic algorithms (KeyGen, InitC, InitM, Refund, Refute, Resolve) and two interactive protocols (Establish, Pay). These are defined in Figure 2. For completeness we also define an optional function Setup($\lambda$) to be run by a trusted party for generating the parameters pp, e.g., a Common Reference
String. In some instantiations the CRS is not required. In this case, we set \( pp := 1 \).

### Using Anonymous Payment Channels

An anonymous payment channel scheme must be used in combination with a payment network capable of conditionally escrowing funds and binding these escrow transactions funds to some data. Such payment networks can be constructed using blockchain-based systems, although they can be built from other technology as well. In this work we assume only the existence of a payment network with these capabilities, and leave the details of the payment network’s implementation (e.g., modeling a blockchain) as a separate problem. We now describe how these algorithms and protocols are used to establish a channel on a payment network.

To instantiate an anonymous payment channel, the merchant \( M \) first generates a long-lived keypair \( (pk_M, sk_M) \leftarrow \text{KeyGen}(pp) \) that will identify it to all customers. The merchant initializes its state \( S \leftarrow 0 \). A customer \( C \) generates an ephemeral keypair \( (pk_C, sk_C) \) for use on a single channel. The customer and merchant agree on their respective initial channel balances \( B_0^{\text{cust}}, B_0^{\text{merch}} \). They now perform the following steps:

1. Each party executes the \( \text{Init}_C \) algorithm on the agreed initial channel balances, in order to derive the channel tokens \( \tau_{C, T_M} \).
2. The two parties transmit these tokens to the payment network along with a transaction to escrow the appropriate funds.
3. Once the funds have been verifiably escrowed, the two parties run the \( \text{Establish} \) protocol to establish the payment channel. If the parties disagree about the initial channel balances, this protocol returns \( \bot \) and the parties may close the channel.
4. If channel establishment succeeds, the customer initiates the \( \text{Pay} \) protocol as many times as desired, until one or both parties close the channel.
5. If the customer wishes to close the channel, she runs \( \text{Refund} \) and transmits \( \tau_{C} \) along with the channel identifier to the payment network.
6. The merchant runs \( \text{Refute} \) on the customer’s closure token to obtain the merchant closure token \( \tau_{M} \).

At the conclusion of this process, the network runs the \( \text{Resolve} \) algorithm to determine the final channel balance and allows each party to collect the determined share of the escrowed funds.

#### 2.2 Correctness and Security

We now described the correctness and security of an anonymous payment channel scheme. Here we provide intuition, and present formal definitions in Appendix B.

**Correctness.** Informally, an APC scheme is correct if for all correctly-generated parameters \( pp \) and opening balances \( B_0^{\text{cust}}, B_0^{\text{merch}} \in [0, \ldots, \text{val}_{\text{max}}] \), every correct (and honest) interaction following the paradigm described above always produces a correct outcome. Namely, each valid execution of the \( \text{Pay} \) protocol produces success, and the final outcome of \( \text{Refute} \) correctly reflects the final channel balance.

**Security.** The security of an Anonymous Payment Channel scheme is defined in terms of two games, which we refer to as payment anonymity and balance. We now provide an informal description of each property, and refer the reader to Appendix B for the formal definitions.

**Payment anonymity.** Intuitively, we require that the merchant, even in collaboration with a set of malicious customers, learns nothing about a customer’s spending pattern beyond the information that is available outside of the protocol. In our anonymity definition, which extends a definition of Camenisch et al. [17], the merchant interacts with either (1) a series of oracles implementing the real world protocols for customers \( C_1, \ldots, C_N \), or (2) with a simulator \( S \) that performs the customer’s part of the Pay protocol. In the latter experiment, we assume a simulator that has access to side information not normally available to participants in the real protocol, e.g., a simulation trapdoor or control of a random oracle. We require that the simulator has the ability to simulate any customer without access to the customer’s wallet, and without knowing the identity of the customer being simulated. Our definition holds if no adversary can determine whether she is in world (1) or (2). We stress that this definition implies anonymity because the simulator has no information about which party it is simulating.

**Balance.** The balance property consists of two separate games, one for the merchant and one for the customer. In both cases, assuming honest execution of the \( \text{Resolve} \) protocol, this property ensures that no colluding set of adversarial counterparties can extract more value from a channel than justified by (1) the party’s initial channel funding, combined with (2) the set of legitimate payments made to (or by) the adversary. Because the merchant and customer have different interfaces, we define this property in terms of two slightly different games. In each game, the adversarial customer (resp. merchant) is given access to oracles that play the role of the merchant (resp. customer), and allows the parties to establish an arbitrary number of channels with chosen initial balances. The adversary may then initiate (resp. cause the other party to initiate) the Pay protocol repeatedly on adversarially-chosen payment amounts \( \epsilon \). Finally, the adversary can initiate channel closure with the counter-party to obtain channel closure messages \( \tau_{CC}, \tau_{CM} \). The adversary wins if the output of the \( \text{Resolve} \) protocol is inconsistent with the total value funded and paid.

### 3 TECHNICAL PRELIMINARIES

In this section we recall some basic building blocks that we will use in our constructions.

**Commitment schemes.** Let \( \Pi_{\text{commit}} = (\text{CSetup}, \text{Commit}, \text{Decommit}) \) be a commitment scheme where \( \text{CSetup} \) generates public parameters; on input parameters, a message \( M \), and random coins \( r \), \( \text{Commit} \) outputs a commitment \( C \); and \( \text{Decommit} \) on input parameters and a tuple \( (C, m, r) \) outputs 1 if \( C \) is a valid commitment to the message, or 0 otherwise. In our instantiations, we recommend using the Pedersen commitment scheme [44] based on the discrete logarithm assumption in a cyclic group.
Key generation and channel initialization algorithms:

KeyGen((pp)). This algorithm generates a keypair (pk, sk) for use by each customer or merchant.
Inst(C, pp, Csk, pk). For C ∈ [C, M] this algorithm is run by each party prior to opening a channel. On input the initial channel balances, public parameters and the party’s keypair, the InstC algorithm outputs the party’s channel token TC and a corresponding secret cskC.

Two-party protocols run between a customer C and a merchant M:

- Establish((C((pp, TC, cskC)), M((pp, TC, cskM))). On input public parameters and each of the initial channel tokens, the Establish protocol activates a channel between two parties who have previously escrowed funds. If successful, the merchant receives established and the customer receives a wallet w. Either party may receive the distinguished failure symbol ⊥.
- Pay((C((pp, CskC)), M((pp, CskM))). On input parameters, a payment amount e, and a wallet wM from a customer, and the merchant’s current state SM (initially 0), the merchant receives a payment success bit RC and a new wallet wM, if the interaction succeeded. The merchant receives a payment success bit RM and an updated state SM if the interaction succeeded.

Channel closure and dispute algorithms, run by the customer and merchant respectively:

- Refund(pp, TC, cskC, w). On input a wallet w, outputs a customer channel closure message rC.
- Refute(pp, TC, S, rC). On input the merchant’s current state SM and a customer closure message, outputs a merchant channel closure message rM and an updated merchant state SM.

- Dispute resolution algorithm, run by the network:
  - Resolve((pp, TC, TC, rC, rM)). On input the customer and merchant’s tokens TC, TC along with closure messages rC, rM (where either message may be null), this algorithm outputs the final channel balance Tfinal C, Sfinal C.

![Figure 2: Definition of an Anonymous Payment Channel scheme.](image)

Symmetric encryption schemes. Our constructions require an efficient symmetric encryption scheme as well as a one-time symmetric encryption scheme. We define a symmetric encryption scheme Πsymenc = (SymKeyGen, SymEnc, SymDec) where SymKeyGen outputs a k-bit key. We also make use of a one-time encryption scheme Πotenc = (OTKeyGen, OTEnc, OTDec). In practice, the encryption scheme can be implemented by encoding the plaintext as an element in a cyclic group G and multiplying by a random group element. In either case, our constructions require that the schemes provide IND-CPA security.

Pseudorandom Functions. Our unidirectional construction requires a pseudorandom function (PRF) F that supports efficient proofs of knowledge. For our purposes it is sufficient that the PRF be secure for a poly-size input space. In addition to the standard pseudorandomness property, our protocols require that the PRF should also possess a property we refer to as strong pre-image resistance. This property holds that, given access to an oracle implementing the function F, for a random seed s, no adversary can find an input point x and a pair (s’, x’) in the domain of the function such that F(s)(x) = F(s’)(x’) except with negligible probability. We propose to instantiate F using the Dodis-Yampolskiy PRF [31], the public parameters are a group G of prime order q with generator g. The seed is a random vector s ∈ Zq and the function is computed as F(s)(x) = g^1/(s’x) for x in a polynomially-sized set. We show in the full version of this paper [33] that the Dodis-Yampolskiy PRF satisfies the strong pre-image resistance property.

Signatures with Efficient Protocols. Our schemes make use of a signature scheme Πsig = (SigKeygen, Sig, Verify) with efficient protocols, as proposed by Camenisch and Lysyanskaya [18]. These schemes feature: (1) a protocol for a user to obtain a signature on the value(s) in a commitment without the signer learning anything about the message(s), and (2) a protocol for (non-interactively) proving knowledge of a signature. Several instantiations of these signatures have been proposed in the literature, including constructions based on the Strong RSA assumption [18] and various assumptions in bilinear groups [8, 19]. For security, we assume that all signatures satisfy the property of existential unforgeability under chosen message attack (EU-CMA).

Non-Interactive Zero-Knowledge Proofs. We use several standard results for non-interactively proving statements about committed values, such as (1) a proof of knowledge of a committed value, and (2) a proof that a committed value is in a range. When referring to the proofs above, we will use the notation of Camenisch and Stadler [22]. For instance, PoK{R(x, r) : y = g^x h^r \land (1 \leq x \leq n)} denotes a zero-knowledge proof of knowledge of integers x and r such that y = g^x h^r holds and 1 ≤ x ≤ n. All values not in enclosed in ( )’s are assumed to be known to the verifier. Our protocols require a proof system that provides simulation extractability, which implies that there exists an efficient proof extractor that (under specific circumstances, such as the use of a simulation CRS) can extract the witness used by an adversary to construct a proof, even when the adversary is also supplied with simulated proofs. In practice we can conduct these proofs non-interactively using a variety of efficient proof techniques [8, 12, 14, 16, 20, 27, 34, 35, 48].

4 PROTOCOLS

In this section we present our main contribution, which consists of three protocols for implementing anonymous payment channels. Our first protocol in §4.1 is a unidirectional payment channel based on e-cash techniques. Our second construction in §4.2 allows for bidirectional payments, with a more complex protocol for handling aborts. Finally, in §4.3 we propose an approach for third-party payments, in which two parties transmit payment via an intermediary.

4.1 Unidirectional payment channels

Our first construction modifies the compact e-cash construction of Camenisch et al. [17] to achieve efficient and succinct unidirectional payment channels. We now provide a brief overview of this construction.
Compact e-cash. In a compact e-cash scheme, a customer withdraws a fixed-size wallet capable of generating B coins. The customer’s wallet is based on a tuple \((k, sk, B)\): \(k\) is an (interactively generated) seed for a pseudorandom function \(F\), \(sk\) is the customer’s private key, and \(B\) is the number of coins in the wallet. Once signed by the merchant, this wallet can be used to generate up to \(B\) coins as follows: the \(i^{th}\) coin consists of a tuple \((s, T, \pi)\) where \(s\) is a “serial number” computed as \(s = F_k(i)\); \(T\) is a “double spend tag” computed such that, if the same coin is spent twice, the double spend tags can be combined to reveal the customer’s key \(pk\) (or \(sk\)); and \(\pi\) is a non-interactive zero-knowledge proof of the following statements:

1. \(0 < i \leq B\)
2. The prover knows \(sk\).
3. The prover has a signature on the wallet \((k, sk, B)\).
4. The pair \((s, T)\) is correctly structured with respect to the signed wallet.

This construction ensures that double spending is immediately detected by a verifier, since both transactions will share the serial number \(s\).\(^7\) The verifier can then recover the spender’s public key by combining the double-spend tags. At the same time, the individual coin spends cannot be linked to each other or to the user. Camenisch et al.\(^{[17]}\) show how to construct the proof \(\pi\) efficiently using signatures and proof techniques secure under the Strong RSA or bilinear assumptions in the random oracle model. Subsequent work presents efficient proofs in the standard model\(^{[8, 9]}\).

Achieving succinct closure. Let us recall our intuition for using compact e-cash in a unidirectional payment channel (see §1.3). In this proposal, the merchant plays the role of the bank and issues the customer a wallet of \(B\) coins, which she can then (anonymously) spend back to the merchant. To close a channel, the customer simply provides the merchant with a series of signed ciphertexts \((C_1, \ldots, C_B)\) each of which contains a coin spend tuple of the form \((s, T, \pi')\) where \(\pi'\) is identical to the normal compact e-cash proof, but simply proves that \(s, T\) are correct with respect to \(wCom\) which is not yet signed by the merchant. These ciphertexts are structured so that a key revealed for the \(j^{th}\) ciphertext will also open each subsequent ciphertext.

The key feature of this approach is that the merchant does not need to know if these ciphertexts truly contain valid proofs at the time the channel is opened. To reveal the remaining \(j\) coins in a channel, the customer reveals a key for the \(j^{th}\) ciphertext, which allows the merchant to “unlock” all of the remaining coin spends and verify them with respect to the commitment \(wCom\) embedded in the customer’s channel token. If any ciphertext fails to open, or if the enclosed proof is not valid, the merchant can easily prove malfeasance by the customer and obtain the balance of the channel. This requires only symmetric encryption and a means to “chain” symmetric encryption keys – both of which can easily be constructed from standard building blocks.\(^9\) Our schemes additionally require a one-time encryption algorithm \(OTEnc\) where the keyspace of the algorithm is also the range of the pseudorandom function \(F\).

We now present the full scheme:

Setup\((1^\lambda)\). On input \(\lambda\), optionally generate CRS parameters for (1) a secure commitment scheme and (2) a non-interactive zero knowledge proof system. Output these as \(pp\).

\[\text{KeyGen}(pp). \text{Compute } (pk, sk) \leftarrow \Pi_{\text{sig}} \cdot \text{SigKeygen}(1^\lambda).\]

\[\text{Init}_C(pp, \text{B}_{\text{cunt}}, \text{B}_{\text{merch}}, pk_{C}, sk_{C}).\] On input a keypair \((pk_{C}, sk_{C})\), uniformly sample two distinct PRF seeds \(k_1, k_2\) and random coins \(r\) for the commitment scheme. Compute \(wCom = \text{Commit}(sk_{C}, k_1, k_2, \text{B}_{\text{cunt}}; r)\). For \(i = 1\) to \(B\), sample \(ck_{i} \leftarrow \text{SymKeyGen}(1^\lambda)\) to form the vector \(ck\). Output \(T_C = (wCom, pk_{C})\) and \(csk_{C} = \langle sk_{C}, k_1, k_2, r, \text{B}_{\text{cunt}}, ck \rangle\).

\[\text{Init}_M(pp, \text{B}_{\text{cunt}}, \text{B}_{\text{merch}}, pk_{M}, sk_{M}).\] Output \(T_M = pk_{M}, csk_{M} = \langle sk_{M}, \text{B}_{\text{cunt}} \rangle\).

\[\text{Refund}(pp, T_M, csk_{C}, w).\] Parse \(w\) (generated by the Establish and Pay protocols) to obtain \(\hat{ck}\) and the current coin index \(i\). Compute \(\sigma \leftarrow \text{Sign}(sk_{C}, \text{refund}[\text{cID}[i][\hat{ck}])\) (where \(\text{cID}\) uniquely identifies the channel being closed) and output \(\text{rC}_C := \langle \text{cID}, i, \hat{ck}, \sigma \rangle\).

\[\text{Refute}(pp, T_C, S, \text{rC}_C).\] Parse the customer’s channel closure message \(\text{rC}_C\) as \((\text{cID}, i, \hat{ck}, \sigma)\) and verify \(\text{cID}\) and the signature \(\sigma\). If the signature verifies, then obtain the ciphertexts \(C_1, \ldots, C_B\) stored after the Establish protocol. For \(i = i\) to \(B\), compute \((\bar{c})[\hat{c}][\hat{c}][\hat{c}][\hat{c}]\left(\text{SymDec}(\hat{ck}, C_i)\right)\) and verify the signature \(\hat{\sigma}\) and the proof \(\hat{\pi}\). If (1) the signature \(\hat{\sigma}\) or the proof \(\hat{\pi}\) fail

\(^7\)In the original compact e-cash construction\(^{[17]}\), the key \(k\) was generated using an interactive protocol between the customer and bank, such that honest behavior by one party ensured that \(k\) was uniformly random. In our revised protocol below, \(k\) will be chosen only by the customer. This does not enable double-spending, provided that the PRF is deterministic and the proof system is sound.

\(^8\)Indeed, an alternative proposal is to construct the coin serial numbers using a chained construction, where each \(s_i\) is computed as a one-way hash of the key used in the previous transaction. This would allow the customer to revoke the channel by posting a secret from one transaction. Unfortunately, proving the correctness of \(s_i\) using standard zero-knowledge techniques would then require \(O(B)\) proving cost, and moreover, does not seem easy to accomplish using the efficient zero knowledge proof techniques we recommend in this work.

\(^9\)For example, the necessary properties can be achieved using a secure commitment scheme and any secure symmetric encryption mechanism.
to verify, (2) any ciphertext fails to decrypt correctly, or (3) any of the decrypted values \((s_j, t_j)\) match a valid spend containing \((s_j, t_j)\) in \(S\). When OTDec \((u_j, t_j) = pk_c: record the invalid result into \(r_c_M\) along with \(cI_d\) and sign the result using \(sk_m\) so that it can be verified by the network. Otherwise set \(r_c_M = (\text{accept})\) and sign with \(sk_m\). Finally, for each valid \(C_j,\) set \(S \leftarrow S \cup (s_j, t_j, \pi)\) and output \(S\) as the new merchant state.

Resolve(pp, \(T_c, T_m, r_c C, r_c M\)). Parse the customer and merchant closure messages and verify all signatures. If any fail to verify, grant the balance of the channel to the opposing party. If \(r_c C = (N, sk_N, \sigma)\) and \(r_c M = \text{accept}\) then set \(B_0^{\text{final}}\) to \((B_0^{\text{cust}} - N) + 1\). Otherwise, evaluate the merchant closure message to determine whether the customer misbehaved. If so, assign the merchant the full balance of the channel.

We present the Establish and Pay protocols in Figure 3.

4.1.1 Security Analysis.

**Theorem 4.1.** The unidirectional channel scheme satisfies the properties of anonymity and balance under the assumption that (1) \(F\) is pseudorandom and provides strong pre-image resistance, (2) the commitment scheme is secure, (3) the zero-knowledge system is sound and zero-knowledge, (4) the signature scheme is existentially unforgeable under chosen message attack and signature extraction is blind, and (5) the symmetric encryption and one-time encryption scheme are each IND-CPA secure.

A proof of Theorem 4.1 can be found in the full version of this paper [33].

4.2 Bidirectional payment channels

The key limitation of the above construction is that it is unidirectional, and only supports payments from a customer to a merchant. Additionally, it supports only fixed-value coins. In this section we describe a construction that enables bidirectional payment channels which feature compact closure, compact wallets, and allow a single run of the Pay protocol to transfer arbitrary values (constrained by a maximum payment amount).

In this construction the customer's wallet is structured similarly to the previous construction: it consists of \(B_0^{\text{cust}}\), and a random wallet public signature key \(wpk\). The customer first commits to these values and sends the resulting commitment to the payment network. The wallet is activated when the customer and merchant interact to provide the customer with a blind signature on its contents.

The key difference from the first protocol is that, instead of conducting the payment \(\epsilon\) using a series of individual coins, each payment has the customer (1) prove that it has a valid signed wallet with balance \(B_0^{\text{cust}} \geq \epsilon\) of currency in it, and (2) request a blind signature on a new wallet for the amount \(B_0^{\text{cust}} - \epsilon\) (and embedding a fresh wallet public key \(wpk_{\text{new}}\)). Notice that in this construction the value \(\epsilon\) can be positive or negative. The customer uses a zero knowledge proof and signatures with efficient protocols to prove that the contents of the new requested wallet are constructed properly, that the balances of the new wallet differs from the original balance by \(\epsilon\), and that \((B_0^{\text{cust}} - \epsilon) \geq 0\). At the conclusion of the transaction, the customer reveals \(wpk_{\text{old}}\) to assure the merchant that this wallet cannot be spent a second time, and the old wallet is invalidated by the customer signing a "revoked" message with \(wsk\) the corresponding private key. Closing the channel consists of the customer posting a valid wallet signed by the merchant to the blockchain. The challenge in this construction is to simultaneously invalidate the existing wallet and sign the new one. If the merchant signs the new wallet before the old wallet is invalidated, then the customer can retain funds in the old wallet while continuing to use the new one. On the other hand, if the merchant can invalidate the old wallet before signing the new one, the customer has no way to close the channel if the merchant refuses to sign the new wallet.

To solve this, we separate the wallet — used in interactive payments — from the value that is posted to perform channel closure and use a two phase protocol to obtain each of these values. Instead of revealing most recent wallet \(w, C\) closes the channel using a refund token \(rt\) which specifies \(B_0^{\text{cust}}\), the current wallet’s public key, and a signature by the merchant. In phase one of Pay, the customer first obtains a signature on the refund token blindly from \(M\). In the second phase, the customer invalidates the old wallet, and then the merchant signs the new wallet. If the merchant refuses to sign the new wallet, the customer can safely close the channel using \(rt\).

We now describe the revised scheme. The protocols Establish and Pay are presented in Figure 4. The Setup and Init algorithms are identical to the previous construction.

- **KeyGen(pp).** Compute \((pk, sk) \leftarrow \Pi_{\text{sig}} \text{SigKeyGen}(\lambda^3)\).
- **InitC(pp, cI_d, B_0^{\text{cust}}, B_0^{\text{mech}}, pk_c, sk_c).** The customer generates the wallet commitment by sampling random coins \(r\), computing an ephemeral keypair \((wpk, wsk) \leftarrow \text{KeyGen}(pp)\) and producing a commitment \(wCom = \text{Commit}(cI_d, wpk, B_0^{\text{cust}}, r)\). It outputs the token \(T_c = (pk_c, wCom)\) and retains the secrets \(csk_c = (wCom, sk_c, cI_d, wpk, wsk, r, B_0^{\text{cust}})\).
- **InitM(pp, B_0^{\text{cust}}, B_0^{\text{mech}}, pk_m, sk_m).** Output \(T_m = pk_m, csk_m = (sk_m, B_0^{\text{cust}})\).
- **Refund(pp, T_C, csk_C, w).** If the customer has not yet invoked the Pay protocol, it sets \(m := \text{(refundUnsigned, cI_d, wpk, B), r}\). Otherwise set \(m := \text{(refundToken, (cI_d, wpk, B), r, rt)}\). Compute \(\sigma = \text{Sign}(sk_c, m)\). Output \(rc_C = (m, \sigma)\).
- **Refute(pp, T_C, S, rc_C).** If a merchant sees a channel closure message, it first parses \(T_C\) to obtain \(pk_c\). If parses \(rc_C\) as \((m, \sigma)\) and verifies the signature \(\sigma\) using \(pk_c\). If this signature verifies, it parses \(m\) to obtain \((cI_d, wpk, B)\) and verifs that \(cI_d\) matches the channel. Finally, if it has previously stored \((wpk, \sigma_{rev})\) in its state \(S\) then it outputs \(r_c_M = ((\text{revoked, } \sigma_{rev}), \sigma)\) where \(\sigma\) is a valid signature on the message \((\text{revoked, } \sigma_{rev})\) under \(sk_m\). Otherwise it adds the new key \(wpk\) to its state \(S\).
- **Resolve(pp, T_C, T_M, rc_C, rc_M).** Verify that both \(rc_C, rc_M\) are correctly signed by the customer and merchant keys \(pk_c\) and \(pk_m\) respectively. Verify that both tokens contain the same \(cI_d\) and this matches the channel identifier from \(T_C, T_M\). If either of the tokens fails this test, replace it with \(\perp\). Let \(B_{\text{total}} = B_0^{\text{cust}} + B_0^{\text{mech}}\). If \(rc_C\) is \(\perp\), simply output all funds to the merchant.

\(^1\)In the special case where the customer has not obtained a signature on the wallet from the merchant (e.g., because the merchant never accepted the channel opening), it can simply post an opening of the wallet commitment.
(1) Parse csk \(C\) as \((pk_c, sk_c, k_1, k_2, r, B)\). Sample \(s_k \in \{0, 1\}^f\).

(2) Generate a proof \(\pi\) that

\[
PK((sk_c, k_1, k_2) : \text{wCom} = \text{Commit}(sk_c, k_1, k_2; r))
\]

(3) For \(j = 1\) to \(B\):

(a) Compute \(s_j = F_k(j), u_j = F_k(j, \pi^j)\) where

\[
\pi^j = PK((sk_c, k_1, k_2, r) : s = F_k(j) \land u = F_k(j))
\]

\(\land \text{wCom} = \text{Commit}(sk_c, k_1, k_2; r)\)

(b) Compute an internal signature \(\delta_j = \text{Sign}(sk_c, \text{spend}||j||s||\pi^j || ck_j)\).

(c) Compute \(C_j = \text{SymEnc}(ck_j, j||s||u || \pi^j || F_k[j || h_{vkj}])\) and an external signature \(\sigma_j = \text{Sign}(sk_c, \text{coinc} || j || C_j)\).

\[
\text{Return } w = (s_k, sk_c, k_1, k_2, r, B, \sigma_w, 1).
\]

Customer(pp, \(p_{TM}, cs_{k_C}\)) Establish

\[
\text{Merchant}((pp, T_C, cs_{k_M})
\]

\[
\text{Return } w_{\text{final}} := (s_k, sk_c, k_1, k_2, r, B, \sigma_w, i + 1).
\]

Customer(pp, e, "\(\text{wmul}\)"

Pay

\[
\text{Merchant}((pp, e, S_{\text{total}})
\]

\[
(s, t, \pi) \rightarrow
\]

\[
\text{Verify } \pi \text{ and that } (s, t) \notin S. \text{ If so, set } S \leftarrow S \cup (s, t, \pi) \text{ and set } R_M \leftarrow 1, \text{ else set } R_M \leftarrow \bot.
\]

Return \(R_M\)

\[
\begin{array}{|c|c|}
\hline
\text{Customer}(pp, T_M, cs_{k_C}) & \text{Establish} \\
\hline
(1) Parse csk \(C\) as \((pk_c, sk_c, k_1, k_2, r, B)\). Sample \(s_k \in \{0, 1\}^f\). & \text{Return } w = (s_k, sk_c, k_1, k_2, r, B, \sigma_w, 1). \\
(2) Generate a proof \(\pi\) that & \\
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Merchant}(pp, T_C, cs_{k_M}) & \text{Return } \text{established}.
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Customer}(pp, e, \"\text{wmul}\") & \text{Pay} \\
\hline
(1) Parse w\text{mul} as \((s_k, sk_c, k_1, k_2, r, B, \sigma_{\text{wx}})\). Abort if \(i \geq B\). & \text{Return } w_{\text{final}} := (s_k, sk_c, k_1, k_2, r, B, \sigma_w, i + 1).
(2) Compute \(s = F_k(i), t \leftarrow \text{OTEnc}(F_k(i), pk_c)\), and a proof & \\
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Merchant}(pp, e, S_{\text{total}}) & \text{Return } R_M \leftarrow 1.
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Verify } \pi \text{ and that } (s, t) \notin S. \text{ If so, set } S \leftarrow S \cup (s, t, \pi) \text{ and set } R_M \leftarrow 1, \text{ else set } R_M \leftarrow \bot.
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Return } R_M \\
\hline
\end{array}
\]

**Figure 3:** Establishment and Payment protocols for the Unidirectional Payment Channel scheme.

1. Parse \(T_C\) to obtain \((pk_c, \text{wCom})\).
2. Parse \(m\) as \((\text{type}, \text{cID}, \text{wpk}, B)\), \(\text{Token}\).
3. Parse \(m\) as \((\text{revoked}, \text{wpk}, \sigma_{\text{rev}})\). Check that \(\text{Verify}(\text{wpk}, \text{revoked}||\text{cID}||\text{wpk}||\sigma) = 1\). If any check fails, terminate and output \(\text{agent}_{\text{final}} = \text{B}_{\text{total}}\) and \(\text{group}_{\text{final}} = 0\).
4. Perform the following checks:
   a. Check the refund’s validity: If type is refundUnSigned, check that \(\text{wCom} = \text{Commit}(\text{cID}, \text{wpk}, B, \text{Token})\). If the merchant’s token contains \(\sigma_{\text{rev}}\) Otherwise type is refundToken, so check that Token is a valid refund token on \((\text{cID}, \text{wpk}, B)\). If either check fails, terminate and output \(\text{B}_{\text{final}} = \text{B}_{\text{total}}\) and \(\text{group}_{\text{final}} = 0\).
   b. Check the refutation’s validity: and check \(\text{Verify}(\text{wpk}, \text{revoked}||\text{wpk}||\sigma_{\text{rev}}) = 1\). If so, terminate and output \(\text{B}_{\text{final}} = \text{B}_{\text{total}}\) and \(\text{group}_{\text{final}} = \text{B}_{\text{total}} - B\) (i.e. pay the claimed \(B\) to \(C\) and the remainder to \(M\)).

4.2.1 Security Analysis. In §1.3 we noted a main limitation of the bidirectional protocol, namely the possibility that a malicious merchant may abort the protocol. As discussed in that section, this provides only limited advantage to an adversary. Within the context of our security proof we address this in a simpler way, by simply preventing the adversarial merchant from aborting during the Pay protocol.

**Theorem 4.2.** The bidirectional channel scheme satisfies the properties of anonymity and balance under the restriction that the adversary does not abort during the Pay protocol, and the assumption that (1) the commitment scheme is secure, (2) the zero-knowledge system is simulation extractable and zero-knowledge, (3) the blind signature scheme is existentially unforgeable under chosen message attack, and (4) the one time signature scheme is existentially unforgeable under one time chosen message attack.

We sketch a proof of Theorem 4.2 in the full version of this paper [33].

4.3 Bidirectional Third Party Payments

One of the main applications of the bidirectional construction above is to enable third party payments. In these payments, a first party \(A\) makes a payment of some positive value to a second party \(B\) via some intermediary \(I\) with whom both \(A\) and \(B\) have open channels. In this case, we assume that both \(A\) and \(B\) act as the customer for channel establishment, and I plays the role of the merchant. Our goal is that I does not learn the identities of the participants, or the amount being transferred (outside of side information she can learn from her channel state), nor should she trusted to safeguard the participants’ funds. This construction stands in contrast to existing
non-anonymous payment channel schemes [30, 45] where given the chain A \rightarrow I \rightarrow B, the intermediary always learns both the amount and the participants.

The challenge in chaining payment channels is to make the payments atomic. That is, the payer A only wants to pay the intermediary I once I has paid the recipient B. But of course this places the intermediary at risk if A fails to complete the payment. Similarly, the payer risks losing her funds to a dishonest intermediary. There is no purely cryptographic solution to this problem, since it’s in essence fair exchange — a problem that has been studied extensively in multi-party protocols. However, there are known techniques for using blockchains to mediate aborts [5, 10]. This is our approach as well.

Recall from §4.2 that the Pay protocol occurs in two phases. The first portion is an exchange of refund tokens that can be used to reclaim escrowed funds. The second phase generates an anonymous wallet for subsequent payments. For a chained transaction from A \rightarrow I \rightarrow B to be secure, we need only ensure that the first phase of both legs completes or fails atomically.

We accomplish this by adding conditions to the refund tokens. These conditions are simple statements for the network to evaluate on examining a token during the Resolve protocol. Specifically, to prevent the recipient B from claiming funds from I if the payer A has not delivered a corresponding payment to I, we introduce the following conditions into B’s refund token:

1. B must produce a revocation message (i.e. a signature using A’s wsk) on A’s previous wallet.
2. A has not posted a revocation token containing wsk to the ledger.

By condition (1), once B forces a payment on I \rightarrow B, A \rightarrow I cannot be reversed since I has the revocation token. By condition (2) if A \rightarrow I has been already been reversed, B cannot force the payment I \rightarrow B since wsk is already on the ledger.

**Hiding the payment amount.** Our third-party payment construction also provides an additional useful feature. Since I acts only a passive participant in the transaction and does not maintain state for either channel, there is no need for I to learn the amount being paid. Provided that both A and B agree on an amount ε (i.e., both parties have sufficient funds in each of their channels), neither party need reveal ε to I. I need merely be assured that the balance of funds is conserved.

To hide the payment amount, we must modify the proof statement used to construct π₂ from the Pay protocol of Figure 4. Rather...
I bind $\epsilon$ and uses this value as a secret input in computing the payment.

Figure 5. In detail, the steps are as follows:

- **Step 1**: Commitment to a new wallet and a proof $\pi_B$ that the updated balance is greater by $\epsilon$.

- **Step 2**: Commitment to $\epsilon$, $A$ and $B$’s new wallet commitments, proofs $\pi_A$, $\pi_B$, refund token for $A$, conditional refund token for $B$.

- **Step 3**: Revocation token for $A$, conditional refund token for $B$.

- **Step 4**: Signature for $A$’s new wallet, revocation tokens for $A$ and $B$’s old wallets.

- **Step 5**: Signature for $B$’s new wallet.

- **Step 6**: Signature for $A$’s new wallet.

A can then prove to $I$ that the two payments cancel or (if $fee$ is non-zero), leave $B$ with $fee$ extra funds via a proof:

$$\pi_2 = PK((w_{pk}', B, r', \sigma_w, \epsilon, r_e) : wCom' = \text{Commit}(w_{pk}', B - \epsilon; r') \land \text{Verify}(pk_m, (w_{pk}, B, \sigma_w) = 1 \land vCom = \text{Commit}(\epsilon, r_e) \land 0 \leq (B - \epsilon) \leq val_{\text{max}})$$

The protocol. We now combine the process of updating both $A$ and $B$’s wallet into a single protocol flow, which we outline in Figure 5. In detail, the steps are as follows:

1. **Step 1**: $B$ commits to $\epsilon$ and conducts the first move of the variable payment Pay protocol (Figure 4) (with the modified balance-hiding proof described above) and sends a commitment to its new wallet state $wCom'$ to $A$. It sends a proof of correctness for the wallet, $\pi_B$, and commitment randomness to $A$.

2. **Step 2**: $A$ completes its own first move, generating $wCom'_A$, $\pi_A$ and additionally computes $\pi_A$ attesting to the correct state of its original wallet and new wallet commitment. It sends these and $B$’s new wallet commitment and $\pi_A$ to $I$.

3. **Step 3**: $I$, after validating the proofs, issues $A$ a refund token for its new wallet $rt_{\text{ref}A}$ and $B$ a conditional refund token $cr_{\text{ref}B}$ as its new wallet. This token embeds the condition that $B$ must produce a revocation token for $A$’s old wallet and that $A$ must not have closed the channel already.

4. **Step 4**: $A$ completes its second move in the variable payment Pay protocol to generate $\sigma_{\text{ref}A}$, the revocation token for its old wallet. It sends that and the $cr_{\text{ref}B}$ to $B$.

5. **Step 5**: $B$ completes its second move to generate $\sigma_{\text{ref}B}$, the revocation token for its old wallet. After validating that it now has a valid refund token by verifying $\sigma_{\text{ref}B}$, it sends $\sigma_{\text{ref}A}$, $\sigma_{\text{ref}B}$, to $I$.

6. **Step 6**: $I$ completes the remaining moves of the variable payment Pay protocol with $A$ and $B$ individually, giving each a blind signature on their new wallets.

Security and abort conditions. We provide a proof sketch for balance preservation in the full version of the paper [33].

In terms of anonymity, the execution of this protocol is no different in terms of the information revealed than two in parallel payments from $A \rightarrow I$ and $I \rightarrow B$. Our payment anonymity definition already allows this type of attack even for the two party case.

The main challenge in realizing this construction is the possibility that a malicious $I$ can selectively abort the protocol during a transaction. This does not allow $I$ to steal funds, but it does force $A$ and $B$ to transmit messages to the network in order to recover their funds. This potentially links the payment attempt to $A$ and $B$’s channels. Unfortunately, this seems fundamentally difficult to avoid in an interactive protocol.

We note that the anonymity threat is limited in practice by the fact that the channels themselves can be funded with an anonymous currency (e.g., [29, 41, 47]), so linking two separate channels does not reveal the participant identifiers. More importantly, since the intermediary can use this abort technique only one time during the lifetime of a channel, there is no possibility for the merchant to link separate payments on the same channel. Finally, an intermediary who performs this abort technique will produce public evidence on the network, which allows participants to avoid the malicious intermediary.
4.4 From Third Party Payments to Payment Networks

It should be possible to extend the above protocol to allow payments of the form \( A \rightarrow I \rightarrow \ldots \rightarrow I_n \rightarrow B \) via techniques similar to those used in non-anonymous payment channel networks [45]. As discussed in §1.2, it is not possible to hide channel balances in such a setting. The general approach is as follows: we use “hash locks” to enforce that either all refund and revocation tokens are valid or none are. Specifically, we attach to both the fund and revocation tokens a condition that they can only be used if one party reveals \( x \) such that \( y = \mathcal{H}(x) \), where \( x \) is picked by \( A \). Because if one party releases \( x \), all parties may close their channels, this forces the entire sequence of payments to either go through or not. As with Lightning, the timeouts for each channel must be carefully chosen. We leave the exact details of this approach to future work.

4.5 Hiding Channel Balances

Each of the constructions presented above has a privacy limitation: the balance of each payment channel is revealed when a channel is closed. While individuals can protect their identities and initial channel balances by using an anonymous currency mechanism to fund channels, the information about channel balances leaks useful information to the network. We note, however, that in the case of non-disputed channel closure, even this information can be hidden from the public as follows. On channel closure, the customer posts a commitment to the final channel balance, along with a zero-knowledge proof that she possesses a valid channel closure token (i.e., a signature on the channel balance in our bidirectional construction). In systems such as Zerocash [47], the final payment redeeming coins to the merchant and customer can be modified to include an additional statement: the amounts paid in this transaction are consistent with the commitment, and do not exceed the initial funding transaction that created the channel. We leave the precise details of such a construction to future work.

5 IMPLEMENTATION OF THE BIDIRECTIONAL SCHEME

We now detail the integration of Bolt into a cryptocurrency and performance and cryptographic details of a concrete instantiation.

5.1 Integration with a Currency

In this section we consider the problem of integrating the bidirectional Bolt protocol into a Bitcoin like cryptocurrency in a soft fork: a protocol change which does not break backward compatibility with existing nodes. Recall that the bidirectional scheme requires that the channels be funded anonymously in order to protect against aborts linking the aborted payment to the channel opening (this does not hold if one wishes merely to prevent multiple payments on the same channel from being linked together). In these conditions, the anonymity of the payment channel is no better than the anonymity of the underlying cryptocurrency. Of the Bitcoin derived currencies, Zerocash and ZCash [3, 47] provide a strong underlying anonymity layer. Anonymity tools for Bitcoin, such as Coinjoin [39], may also be sufficient in some circumstances and future improvements to Bitcoin may increase the achievable anonymity. The mechanism for deployment is compatible with either currency.

In Bitcoin and ZCash each transaction\(^\text{12}\) consists of a set of inputs and a set of outputs. Inputs refer to a previous transaction output and contain a ScriptSig authorizing use of the funds. Outputs specify the amount of the output and a ScriptPubKey specifying when the output can be spent. To evaluate a transaction the ScriptPubKey from the previous transaction and ScriptSig from the current transaction are combined and evaluated using a stack-based scripting language. In the simplest case, ScriptPubKey requires a signature under a specified public key to spend the funds and ScriptSig contains such a signature. However, more complex scripts are allowed including control flow such as if statements, time locks that enforce that a given number of blocks has elapsed since the transaction was created, and threshold signatures. As long as the combined script evaluates to True, spending is authorized.

Our soft fork approach involves adding a single opcode, OP_BOLT to the scripting language. This opcode has the power to (1) validate the commitment opening and blind signature on the commitment in a refund token, and (2) inspect the output of the transaction and enforce restrictions on it.

Most opcodes do not inspect transaction outputs. The notable exception to this rule are signature opcodes that may hash the entire transaction, including both inputs and outputs, in order to verify the signature. However, it is entirely possible to modify the ZCash and Bitcoin codebase to enable opcodes that do have access to transaction outputs in general.\(^\text{13}\) Specifically, our new opcode will enforce two constraints on the outputs of a transaction closing the channel:

1. Verifying that there are two outputs: one paying the merchant his balance and the other paying the customer hers.
2. Verifying that the customer’s payout is time locked such that it can be claimed by the merchant if the refund token has been invalidated (i.e. the customer tried to close on an old channel state).

To accomplish this, we implement the Pay protocol so that the revocation token is the private key corresponding to a Bitcoin (resp. transparent ZCash) address. When a channel is closed, the merchant’s fraction of the channel balance is paid in a transaction that is spendable immediately. However, the customer’s funds are not immediately spendable by the customer since we need to allow the merchant to dispute the closure. The merchant does this by signing a transaction with both his key and the revocation token key. If the merchant does not do so, the customer can claim the funds after some elapsed time. The script is given below:

```markdown
OP_IF # If merchant
    OP_2 <rev-pubkey><merchant-pubkey> OP_2
    OP_CHECKMULTISIG #2 of 2 multi-sig check
OP_ELSE # If customer wait
    <delay> # delay to wait
```

\(^\text{12}\)We refer here to the “unshielded” transactions in ZCash. Shielded transactions function differently.

\(^\text{13}\)In systems derived from Bitcoin core’s source code, this requires some modification as the current architecture abstraction in script evaluation provides a callback to get the transaction hash, not direct access to the transaction itself. However this is not a protocol limitation and indeed past versions of the codebase did expose direct access. Exposing direct access does not affect consensus rules in and of itself.
Merchant
Setup(ms) Establish(ms) Pay(ms)
1433.51 ± 23.69 15.87 ± 0.27 82.32 ± 1.37
Algebraic MACs[24]
6.90 ± 0.17 37.61 ± 0.93

Figure 6: Performance comparison of different implementations of BOLT bidirectional payment protocol. 1000 iterations on a single core of an Intel(R) Xeon(R) CPU E5-2695 v4 @ 2.10GHz. Customer setup is included in Establish.

<table>
<thead>
<tr>
<th>primitive</th>
<th>Customer Establish(ms)</th>
<th>Pay(ms)</th>
<th>Merchant Establish(ms)</th>
<th>Pay(ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bilinear CL-Sigs[19]</td>
<td>8.07 ± 0.13</td>
<td>100.13 ± 1.60</td>
<td>1433.51 ± 23.69</td>
<td>15.87 ± 0.27</td>
</tr>
<tr>
<td>Algebraic MACs[24]</td>
<td>6.90 ± 0.17</td>
<td>37.61 ± 0.93</td>
<td>826.78 ± 19.26</td>
<td>11.97 ± 0.31</td>
</tr>
</tbody>
</table>

### 6 RELATED WORK

**Anonymity and scaling for Bitcoin.** A number of works have proposed additional privacy protections for Bitcoin. Zerocoin, Zerocash and similar works [41, 47] provide strong anonymity through the use of complex zero knowledge proofs. A separate line of works seek to increase anonymity by Bitcoin by mixing transactions (e.g. CoinJoin [39], CoinShuffle, CoinSwap). Like Bitcoin, each of these constructions require that all transactions are stored on the blockchain. Finally, recent work has proposed probabilistic payments as an alternative payment mechanism [43].

**Privacy in payment channels** As discussed in detail in the introduction, Heilman et al. [37] construct off-chain payments with 3rd party privacy.

**Lightning anonymity limitations.** The Lightning Network [45] does not provide payment anonymity between pairs of channel participants – i.e., a merchant can see the channel identity of every customer that initiates a payment. However, the protocol includes some limited anonymity protections for path payments. These operate on a principle similar to an onion routing network, by using multiple non-colluding intermediaries to obscure the origin and destination of a path. Unfortunately this proposal suffers from collusion problems: given the chain $A \rightarrow I_1 \rightarrow I_2 \rightarrow I_3 \rightarrow B$, only $I_1$ and $I_3$ must collude to recover the identities of $A$ and $B$, since all transactions on the path share the same Hash Timelock Contract ID. Moreover, this security mechanism assumes there exist a network with sufficient path diversity for these protections to be viable.

### 7 ACKNOWLEDGMENTS

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### REFERENCES

We now describe in depth our choice of cryptographic primitives:

A.1 Possible building blocks

Signatures with efficient protocols are the core building block of anonymous credentials and are a well studied primitive with many solutions offering various performance, security, and feature tradeoffs. One of the most efficient schemes that offers a full set of features and provable security is the bilinear variant of CL-signatures due to Camenisch and Lysyanskaya [19]. An implementation exists in Charm [4].

We are aware of two other candidate signature schemes with available implementations from petLib [28] that are aimed at providing increased performance with reduced functionality. The first is used in the construction of Lightweight Anonymous Credentials [7]. Here signatures can only be shown anonymously once. Second, Algebraic MACs are used in [24], to build a limited form of anonymous credential. Because it uses a MAC not a signature, only the issuer can verify “signed” messages. This requires some modification to our protocol since closure of a channel currently requires public verification of the refund token.

A.2 Selecting the signature scheme

The scheme from Anonymous Credentials Light [7] is the fastest for issuing and showing, with most operations taking less than 0.01ms. However, a registration phase must be completed for the set of messages that can be signed. This must be repeated every time the set changes and takes 100ms. Because the refund token $rt$ is selected at random and changes on every instance, this process must be done on every payment. Moreover, even if this were made far faster, the registration process reveals the message set. It may be possible to patch Bolt to accommodate this or modify the credential scheme to remove the restriction, but the 100ms cost is too high to pay per payment. The remaining two schemes are more promising with most operations taking less than 30ms each.
A.3 Implementation
We build two completely distinct implementations of the bidirectional payment protocol. One using bilinear CL-Sigs and the other using Algebraic MACs. Our approach mirrors the construction of a credential scheme: we present a commitment to the wallet and a proof that it is signed and then use Schnorr proofs [48] to prove the balance of the new wallet commitment is correct with respect to the old wallet. We then blindly obtain a signature on the new wallet. For the range proof, we use a technique reminiscent of [16]: we decompose the balance into bytes and prove we have a signature issued on each byte. This allows us to reuse the code and primitives from the signature scheme rather than using a separate range proof which would introduce more cryptographic assumptions, more code, and more dependencies.

The results are given in Table 6. The implementation based on Algebraic MACs is approximately twice as fast as the CL-Sig approach. It should be noted, however, that there is far more room for optimization in the CL-signature library. While both are implemented in Python, the implementation of Algebraic MACs use only elliptic curve operations via openssl. As such, the principle overhead is from calling native code from Python. On the other hand, the CL-signature implementation uses symmetric bilinear pairings with an implementation from the PBC library [38]. Use of asymmetric pairings and a faster pairing library such as RELIC [6] would give a marked improvement.

A.4 Adapting channel closure to avoid public verification of credentials
Closing a disputed channel currently requires the blockchain to verify that the refund token is signed. For our faster construction, this is impossible since the key remains secret and the “signature” is actually a MAC. There are two solutions to this: 1) We can, as outlined in paragraph 5.1 opt to have the blockchain validate conventional signatures at the cost of an extra round trip in pay. 2) We can allow the merchant to prove to the recipient that the MAC is not a forgery.

The MAC itself consists of \( u, u' = uH_u(m) \) where \( H_u(m) \) is a keyed and deterministic hash function. Unfortunately, \( u \) is chosen at random so the MAC is not unique and it is not sufficient to reveal the MAC on the message and prove its correctness.\(^{14}\) Instead, we must prove that \( \log_u(u') \neq \log_u(u') \) (i.e. that \( u_k^{\lambda}(m) \neq u') \) and that the revealed MAC \( v, v' \) is correct. Canemisch and Shoup give an extremely efficient proof for discrete log inequality [21] where only one discrete log is known to the prover and none known to the verifier. We implement this full proof of valid MAC combining the proof of MAC validity and discrete log inequality. It takes approximately 14ms to generate and verify. We note that this proof includes the actual valid MAC on the forged refund token, it is necessary for the blockchain to blacklist this MAC and not accept it. However, since the refund token can never be used in payments, we need not add extra steps to the pay protocol.

\(^{14}\)Counter-intuitively, despite being built on a MAC, Keyed-Verification Anonymity Credentials include an efficient zk-proof of validity of a MAC that effectively transforms the MAC into a (non-blind) signature. Since this proof is somewhat expensive, it is only used to verify the correctness of issued credentials.

B SECURITY DEFINITIONS
In this section we provide formal security definitions for an anonymous payment channel scheme.

B.1 Payment anonymity
Let \( \mathcal{A} \) be an adversary playing the role of merchant. We consider an experiment involving \( P \) "customers", each interacting with the merchant as follows. First, \( \mathcal{A} \) is given pp, then outputs \( T_M \). Next \( \mathcal{A} \) issues the following queries in any order:

- **Initialize channel for \( C_i \)**: When \( \mathcal{A} \) makes this query on input \( B^\text{cust}, B^\text{merch} \), it obtains the commitment \( T_i^C \), generated as \( (T_i^C, csk_i^C) \leftarrow \text{Init}_C(pp, B^\text{cust}, B^\text{merch}) \).
- **Establish channel with \( C_i \)**: In this query, \( \mathcal{A} \) executes the Establish protocol with \( C_i \) as:
  \[ \text{Establish}((C(pp, T_M, csk_i^C)), \langle \mathcal{A}(\text{state}) \rangle) \]
  Where state is the adversary’s state. Let us denote the customer’s output as \( w_i \), where \( w_i \) may be \( \perp \).
- **Payment from \( C_i \)**: In this query, if \( w_i \neq \perp \), then \( \mathcal{A} \) executes the Pay protocol for an amount \( \epsilon \) with \( C_i \) as:
  \[ \text{Pay}((C(pp, \epsilon, w_i)), \langle \mathcal{A}(\text{state}) \rangle) \]
  Where state is the adversary’s state. We denote the customer’s output as \( w_i \), where \( w_i \) may be \( \perp \).
- **Finalize with \( C_i \)**: When \( \mathcal{A} \) makes this query, it obtains the closure message \( r_i^C \), computed as \( r_i^C \leftarrow \text{Refund}(pp, w_i) \).

We say that \( \mathcal{A} \) is legal if \( \mathcal{A} \) never asks to spend from a wallet where \( w_i = \perp \) or where \( w_i \) is undefined, and where \( \mathcal{A} \) never asks \( C_i \) to spend unless the customer has sufficient balance to complete the spend.

Let \( auxparams \) be an auxiliary trapdoor not available to the participants of the real protocol. We require the existence of a simulator \( S^{X-Y}() \) \((pp, auxparams, \cdot) \) such that for all \( T_M \), no allowed adversary \( \mathcal{A} \) can distinguish the two experiments with non-negligible advantage:

- **Real experiment.** In this experiment, all responses are computed as described above.
- **Ideal experiment.** In this experiment, the Commitment, Establishment and Finalize queries are handled using the procedure described above. However, in the Payment query, \( \mathcal{A} \) does not interact with \( C_i \) but instead interacts with \( S^{X-Y}() \) \((pp, auxparams, \cdot) \).

As in [17] we note that this definition preserves anonymity because the simulator \( S \) does not know the identity of the user \( i \) for which he is spending the coin.

B.2 Payment Balance
\( \mathcal{A} \) interacts with a collection of \( P \) honest customers \( C_1, \ldots, C_P \) and \( Q \) honest merchants \( M_1, \ldots, M_Q \). Initialize the counters \( bal_{\mathcal{A}} \leftarrow 0 \), \( claimed_{\mathcal{A}} \leftarrow 0 \). Let \( pp \leftarrow \text{Setup}(1^\lambda) \). For each merchant \( i \in [1, Q] \), at the start of the game let \( (pk_{M_i}, sk_{M_i}) \leftarrow \text{KeyGen}(pp) \). Give \( pp \) and \((pk_{M_i}, \ldots, pk_{M_Q})\) to \( \mathcal{A} \). Now \( \mathcal{A} \) may issue the queries described in Figure 7 in any order.

We say that \( \mathcal{A} \) is legal if \( \mathcal{A} \) never asks to spend from a wallet where \( w_i = \perp \) or where \( w_i \) is undefined, and where \( \mathcal{A} \) never asks
**Initialize channel for** $C_i$ (**resp.** $M_i$) **When** $A$ **makes this query on input** $(P_i, B_{0^\text{cust}}, B_{0^\text{merch}})$, **it obtains the commitment** $T_{C_i}$ (**resp.** $T_{M_i}$) **computed as follows:**

- **If the party** $P_i$ **is a customer:** First compute $(pk_{C_i}, sk_{C_i}) \leftarrow \text{KeyGen}(pp)$, then $(T_{C_i}, cs_k l_{C_i}) \leftarrow \text{Init}_{C_i}(pp, B_{0^\text{cust}}, B_{0^\text{merch}}, pk_{C_i}, sk_{C_i})$. Set $\text{bal}_A \leftarrow \text{bal}_A + B_{0^\text{cust}}$.

- **If the party** $P_i$ **is a merchant:** Compute $(T_{M_i}, cs_k l_{M_i}) \leftarrow \text{Init}_{M_i}(pp, B_{0^\text{cust}}, B_{0^\text{merch}}, pk_{M_i}, sk_{M_i})$. Set $\text{bal}_A \leftarrow \text{bal}_A + B_{0^\text{cust}}$.

- **Return a unique channel identifier** corresponding to the resulting channel, and store sid as well as the secrets for each party.

**Establish channel with** $C_i$ (**resp.** $M_i$). **When** $A$ **specifies** $(P_i, \text{sid}, T_A)$, **and** $A$ **has previously initialized a channel labeled** sid **with party** $P_i$, **recover the secret information associated with the party and channel label, then execute the Establish protocol with** $C_i$ (**resp.** $M_i$) **using the following input:**

- **If** $P_i$ **is a customer:** Establish($[(C_i, pp, T_A, cs_k l_{C_i})]$, $[A(\text{state})] \rightarrow w_i$ (**or** $\perp$).

- **If** $P_i$ **is a merchant:** Establish($[(A(\text{state})], M_i(p, T_A, cs_k l_{C_i})] \rightarrow \text{established}$ (**or** $\perp$).

Where $\text{state}$ **is the adversary’s state.**

**Payment from** $C_i$ (**resp.** $M_i$). **In this query,** $A$ **specifies** $(P_i, \text{sid}, \epsilon)$ **where** $\epsilon$ **may be positive or negative. If** $A$ **has previously conducted the Establish protocol for channel** sid **with this party and the party’s output was not $\perp$, then execute the Pay protocol with** $A$ **as:**

- **If** $P_i$ **is a customer:** Pay($[(C_i(p, \epsilon, w_i)], [A(\text{state})]) \rightarrow w_i$ (**or** $\perp$). If the customer’s output is not $\perp$, set $\text{bal}_A \leftarrow \text{bal}_A + \epsilon$.

- **If** $P_i$ **is a merchant:** Pay($[(A(\text{state})], M_i(p, \epsilon, S_i)) \rightarrow S_i$ (**or** $\perp$). If the merchant’s output is not $\perp$, set $\text{bal}_A \leftarrow \text{bal}_A - \epsilon$.

Where $\text{state}$ **is the adversary’s state.**

**Finalize with** $C_i$ (**resp.** $M_i$). **When** $A$ **makes this query on input** $(P_i, \text{sid})$ **and optional input** $\text{rc}_M$, **if it has previously established a channel labeled** sid **with** $P_i$, **it obtains a closure message as:**

- **If** $P_i$ **is a customer:** if $A$ has previously established a channel with $P_i$ and has not previously Finalized on this party, compute $\text{rc}_C \leftarrow$ 

  - $\text{refund}(pp, w_i)$, store $\text{rc}_C$, and return $\text{rc}_C$ to $A$.

- **If** $P_i$ **is a merchant:** if $A$ has previously established a channel with $P_i$ and has not previously Finalized on this party, compute $\text{rc}_M \leftarrow$ 

  - $\text{Refute}(pp, S_i, \text{rc}_C)$.

If the adversary provided $\text{rc}_M$ and $\text{rc}_C$ is stored, compute $(B_{\text{final}}^\text{merch}, B_{\text{final}}^\text{cust}) \leftarrow \text{Resolve}(pp, T_C, T_M, \text{rc}_C, \text{rc}_M)$ and update $\text{claimed}_A \leftarrow \text{claimed}_A + B_{\text{final}}^\text{merch}$ (**resp.** $B_{\text{final}}^\text{cust}$).

**Figure 7:** Queries for the Payment Balance game.

$C_i$ **to spend unless the customer has sufficient balance to complete the spend. We say that** $A$ **wins the game if at the conclusion of** $A$’s **queries, we have** $\text{claimed}_A > \text{bal}_A$. 

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